

# Exercises

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Here are a few exercises for you about generating subgroups. The subsequent exercises are taken from the book by Magnus, Karrass and Solitar.

## Generating subgroups

Recall that if  $G$  is a group generated by a set  $X$ , if  $H$  is a subgroup of  $G$  and if  $R$  is a right transversal of  $H$  in  $G$ , then

$$H = gp(Y)$$

where

$$Y = \{\delta(r, x) = rx(r\bar{x})^{-1} \mid r \in R, x \in X\}.$$

The method here gives rise to a symbol-by-symbol rewriting procedure. What this means is that if

$$h = x_1^{\epsilon_1} \dots x_n^{\epsilon_n}$$

then we rewrite  $h$  as a  $Y$ -product as follows:

1. if  $\epsilon_j = 1$ , we replace  $x_j$  by

$$y_j = \overline{x_1^{\epsilon_1} \dots x_{j-1}^{\epsilon_{j-1}}} . x_j . \overline{(x_1^{\epsilon_1} \dots x_j^{\epsilon_j})}^{-1};$$

2. if  $\epsilon_j = -1$ , replace  $x_j^{-1}$  by  $z_j^{-1}$  where

$$z_j = \overline{x_1^{\epsilon_1} \dots x_j^{\epsilon_j}} . x_j . \overline{(x_1^{\epsilon_1} \dots x_{j-1}^{\epsilon_{j-1}})}^{-1}.$$

1. Suppose that  $F$  is a free group on  $a$  and  $b$  and that  $\phi$  is the homomorphism of  $F$  onto  $T = \langle t; t^2 = 1 \rangle$  which sends  $a$  and  $b$  to  $t$ . Find generators for the kernel of  $\phi$ .
2. Suppose that  $F$  is a free group on  $a$  and  $b$  and that  $\phi$  is the homomorphism of  $F$  onto  $T = \langle s, t; a^2 = t^2 = (st)^2 = 1 \rangle$  which sends  $a$  to  $s$  and  $b$  to  $t$ . Find generators for the kernel of  $\phi$ .
3. Suppose that  $F$  is a free group on  $a$  and  $b$  and that  $\phi$  is the homomorphism of  $F$  onto  $E = \langle s, t; s^3 = t^2 = (ts)^2 = 1 \rangle$  which sends  $a$  to  $s$  and  $b$  to  $t$ . Find generators for the kernel of  $\phi$ .
4. Suppose that  $F$  is a free group on  $a$  and  $b$  and that  $\phi$  is the homomorphism of  $F$  onto  $E = \langle s, t; st = ts \rangle$  which sends  $a$  to  $s$  and  $b$  to  $t$ . Find generators for the kernel of  $\phi$ .

Here are a selection of exercises from Magnus, Karrass and Solitar.

## Section 1.1

8, 9, 10.

## Section 1.2

7, 8, 9, 12, 13, 20.

## Section 1.3

6, 8, 12, 13, 16.

## Section 1.4

1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 17, 18, 19, 24, 25.

## **Section 1.5**

2, 3, 4, 8, 10.

## **Section 2.1**

2, 4, 5, 6, 7.

**Read Chapter 2 of MKS**